US – 353

IV Semester B.A./B.Sc. Examination, May 2017
(CBCS) (Fresh + Repeaters) (2015 – 16 and Onwards)
MATHEMATICS (Paper – IV)

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Define normal subgroup of a group.

b) If $f : (G, \circ) \to (G', *)$ is a homomorphism then prove that $f(e) = e'$ where e and e' are the identity elements of G and G' respectively.

c) Calculate a_0 in the Fourier series of $f(x) = e^x$ in $(-\pi, \pi)$.

d) Write Taylor's series of the function f(x, y) about the point (a, b).

e) Find $L[1 - 2e^{3t}]$.

f) Find $L^{-1}\left[\frac{s-1}{(s-1)^2+9}\right]$.

g) Find the particular integral of $(D^2 + 1)\, y = \sin 3x$.

h) Reduce the equation $y_2 - 2\tan x\, y_1 + 5y = 0$ to normal form.

PART – B

Answer **one full** question. **(1×15=15)**

2. a) If $f : (Z, +) \to (2Z, +)$ is defined by $f(x) = 2x, \forall x \in Z$, then show that f is an isomorphism.

b) If $f : G \to G'$ be an isomorphism of a group G onto G' then prove that $\text{Ker} f = \{e\}$ if and only if f is one-one.

P.T.O.

c) If H is a subgroup of G and K is a normal subgroup of G then prove that $H \cap K$ is a normal subgroup of G.

OR

3. a) If H is a normal subgroup of G then prove that G/H is a group w.r.t. the binary operation defined by

$H_a \cdot H_b = H_{ab}\ \forall\ H_a, H_b \in G/H$.

b) If $f : G \to G'$ be a homomorphism of a group G onto G′ with Kernel K, then prove that K is a normal subgroup of G.

c) Show that the mapping $f : (R, +) \to (R^+, \bullet)$ defined by $f(x) = e^x,\ \forall x \in R$, is an isomorphism. (R = set of reals and R^+ = set of positive reals).

PART – C

Answer any two full questions. (2×15=30)

4. a) Obtain the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$.

b) Find the half range cosine series of $f(x) = 2x - 1$ in (0, 2).

c) Expand $e^x \sin y$ in powers of x and y upto second degree terms.

OR

5. a) Find the extreme values of the function $f(x, y) = xy(1 - x - y)$.

b) A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimension of the box requiring least material for its construction.

c) Find the Fourier series of $f(x) = 1 - x^2$ in (–1, 1).

6. a) i) Prove that $L[e^{at}] = \dfrac{1}{s-a}$.

ii) Find L [cosh (t). cos (2t)].

b) Express $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 6, & t > 2 \end{cases}$ in terms of unit step function and find L[f(t)].

c) Find $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$.

OR

7. a) Find $L\left[\frac{e^{-at}-e^{-bt}}{t}\right]$.

b) Using convolution theorem find $L^{-1}\left[\frac{1}{(s^2+1)(s-1)}\right]$.

c) Find $L^{-1}\left[\frac{s^2+3}{(s-1)^2(s+2)}\right]$.

PART – D

Answer **one full** question. **(1×15=15)**

8. a) Solve : $(D^2 - 2D + 1)y = \sinh(x)$.

b) Solve : $x^2y_2 - 2x(x+1)y_1 - 2(x+1)y = x^3$ given that x is a part of complementary function.

c) Solve : $(D^2 + 2D + 4)y = e^x \sin x$.

OR

9. a) Solve : $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \sin(\log x)$.

b) Solve : $\frac{dx}{dt} = 3x - 4y, \frac{dy}{dt} = x - y$.

c) Solve : $\frac{d^2y}{dx^2} + 9y = \sec 3x$ by the method of variation of parameters.